Weak Turbulence in Ocean Waves

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LONG-TERM GOALS

Predict and explain the spectral energy density of internal waves and surface gravity waves in the ocean. Study and explain the interactions between internal waves, eddies and mean currents. To achieve these goals, further develop wave turbulence theory that is used to predict the statistical evolution of complex systems.

OBJECTIVES

(A) Apply weak turbulence methodology using rigorous mathematical techniques to identify possible sources of discrepancies between theory and observation of surface gravity and internal waves in the ocean.

(B) Reformulate weak turbulence in a mathematically self consistent way. Obtain theoretical predictions for general statistical quantities, like Probability Distribution Functions and multi-point correlations.

(C) Use the above developments to construct the form of the stationary spectral energy density and other significant statistical properties of wind driven surface gravity waves.

(D) Use weak turbulence theory to predict the stationary spectral energy density and other statistical properties of internal waves in the ocean. Compare the results with experimental observations.

APPROACH

Weak (or wave) turbulence is a universal theory used for the statistical description of an ensemble of weakly interacting waves. It has been used for the description of ocean waves since pioneering works of Hasselmann [1] and Zakharov [2-3].

The key feature of the weak turbulence description is the derivation of the statistical equation for the time evolution of spectral energy density of wave fields. Such an equation is called a kinetic equation. Derivation of the kinetic equation is in turn based on the Hamiltonian structure of the waves in question and multiple time scale expansions of time evolution equations of the statistical averages.

Key assumptions used for the derivation of the kinetic equations are: (i) weak nonlinear interactions between waves, (ii) Random phases of the waves.

The project approach includes

(1) *Setting stage for use of weak turbulence theory*. To rigorously apply weak turbulence formalism to internal waves, we had to develop from scratch Hamiltonian formalism for internal waves. In our Hamiltonian formulation the resulting field equations are *equivalent* to the dynamical equations, whereas previously used approaches relied on a small displacement approximation to derive similar field equations. Therefore our form of internal wave equations preserves all the necessary symmetries of the original dynamical equations, like energy conservation, incompressibility, etc.

(2) *Further development of wave turbulence theory*. Weak turbulence theory in its traditional formation totally ignores the spectral fluctuations, yet important statistical information is contained in these fluctuations. Understanding these fluctuations is of crucial importance to ONR, as these fluctuations may be responsible for "rare" events.

(3) *Application of weak turbulence theory for description of ensembles of nonlinear interacting waves, like ocean surface waves and internal waves*. For example, we have predicted that steady state of the oceanic internal waves is not limited to Garrett-and-Munk, but in fact much richer, and include the whole family of steady-state solutions.

(4) *Close collaboration with observational work to verify the resulting theorie*s. In particular, close collaboration with Dr. Kurt Polzin was instrumental for progress of this project, as Kurt have confirmed that the prediction of (3) above does indeed hold in the ocean.

Principal collaborators on this project are my graduate student Boris Pokorni, Prof. Esteban Tabak from Courant Institute and Dr. Kurt Polzin from WHOI. Furthermore, I collaborate with Prof. Sergey Nazarenko from the University of Warwick (UK) on general wave turbulence methodology. I have also started to work with Prof. Raffaele Ferrari from MIT on impact of wave-wave interactions on tracers transport in the ocean.

WORK COMPLETED

(A) In the previous years we have systematically derived the kinetic equation appropriate for the description of wave-wave interaction of internal waves. We have proved that the internal wave kinetic equation derived in [5-6] has a family of steady state solutions of the form $n(\mathbf{k},m) \propto |\mathbf{k}|\hat{}(-x) m\hat{}(-y(x))$. Here $y(x)$ represents a *curve* in (x,y) plain of steady state solutions of the kinetic equation of internal waves in the ocean. Remarkably, this theoretical curve passes through the high-frequency-high-wave number limit of the Garrett-and-Munk spectrum of internal waves. Therefore we have shown, for the first time that the high-frequency-high-wave number limit of the Garrett-Munk spectrum of internal waves constitutes an exact steady state solution of the weak turbulence kinetic equation. Detailed and systematic review of oceanographic literature, performed in collaboration with Kurt Polzin and Esteban Tabak, reveals that major ocean internal waves observational programs (NATRE, AIWEX, FASINEX, MODE, PATCHEX, SWAPP and IWEX) exhibit a larger degree of variability than one might anticipate for a universal spectrum. Moreover, the deviations from the Garrett-and-Munk power laws form a pattern that is *consistent* with theoretical $y(x)$ curve obtained with our theory. These findings are reported in [9, 14].

(B) Natural question emerging from the finding described in (A) above is the following: what determines which particular solution of the possible $y(x)$ curve is realized in each particular oceanographic setting? We have started to develop a universal theory that would provide such a *solvability condition* that would answer this question. We first considered the case when ocean rotation is the only perturbation to otherwise scale invariant oceanographic settings. We derived simple selection criteria which, somewhat surprisingly, single out Garrett-and-Munk spectrum as a preferred solution [14]. Work is underway to derive selection mechanisms for the perturbations due to wave-vortex and wave-shear interactions. The effects of tides on internal waves are also being considered.

(C) The novel Hamiltonian formalism that we have developed in [6] and rigorous wave-turbulence reformulation developed in [10-13] allows us to study rigorously the interactions between internal waves and voticity and mean-flows fields. In particular the question that is investigated now is the following: how much energy internal waves drain from vortices? In other words, I think that interaction with internal waves provides the main mechanism for friction of ocean vortices. Continuing ONR funding would enable us to achieve further significant progress on these pressing matters.

(D) The work on generalizing wave turbulence to describe the time evolution of general statistical properties of interacting wave systems is continued. We have recently generalized wave turbulence theory to describe the time evolution of the *Probability Distribution Function,* or PDF's of wave systems. We have achieved this goal by inventing "Generalized Random Phase and Amplitude approach". In this approach we assume that wave's amplitudes and phases are independent random variables, and this allows us to derive the closed equation for time evolution of general statistical quantities. Time evolution equation of the PDF allows studying the self consistency of the wave turbulence approach and intermittency in ocean waves [10-13].

(E) Work is continued on numerical modeling the dynamical equations of gravity surface waves. This work is performed together with my graduate student Boris Pokorni. Previously we have verified that the steady state spectra of these waves in a forced-damped case are indeed consistent with predictions of wave turbulence (Zakharov-Filonenko spectrum), and we demonstrated numerically that surface gravity waves on deep fluid are indeed strongly intermittent for low wave numbers (long waves). Now we have found that the numerical evolution of these waves exhibit rather unexpected behavior as described below. Traditional wave turbulence demonstrates that for each wave vector of the system there correspond an oscillation with the frequency given by the renormalized dispersion relation. This simple picture appears to be incorrect: we have found numerically that there are two types of waves that are present in the system for each wave vector: one wave has a frequency given by the linear dispersion relation, and the second wave has much higher frequency. We are now in the process of studying possible implication of this phenomenon for the ocean gravity waves. We have also verified numerically that amplitudes and phases of the waves are indeed independent random variables, as needed for the theory to be applicable. These results are reported in [14].

RESULTS

Previous results:

- The validity of Zakharov's form of the weak turbulence Hamiltonian was reestablished
- The form of the surface-wave Hamiltonian in physical space was established, and it was found to coincide with generalization of Choi's Hamiltonian [4]. The resulting Hamiltonian is simpler and more compact than the one in [3].
- The equivalence of these two Hamiltonian structures was established.
- The canonical Hamiltonian structure for long internal waves in hydrostatic balance in a rotating environment was derived [6].
- The weak turbulence formalism was generalized for a simple model to include kinetic equations arising from near-resonant interaction of triads of waves [7].
- The kinetic equation for the spectral energy density evolution of internal waves was found, and its solution in the high frequency limit was derived [5].
- Internal waves kinetic equation describing non rotating ocean has not one, but rather a full family of steady state solutions. It was shown that the high-frequency-high-wave number limit of the celebrated Garrett-and-Munk spectrum of internal waves in the ocean constitutes *an exact* steady state solution of the kinetic equation of internal waves [9,14].
- The high-frequency-high wave number limit of the observed internal wave spectra were re-analyzed and it was established that the deviation from the Garrett-and-Munk spectrum of internal waves form a pattern [9].
- This pattern is consistent with the family of wave turbulence theory as described above.
- Weak turbulence theory was generalized to include spectral fluctuations of the interacting wave fields. Prediction for the rate of growth of spectral fluctuations was obtained for surface tension waves [8].
- Wave turbulence was reformulated using the Generalized Random Phase and Amplitude approach. Time evolution for the one-pint probability distribution function was derived. It was shown that in the presence of wave breaking in the system the probability flux will be formed in an amplitude space which will lead to strong intermittency (higher-than-Gaussian probability of high-amplitude waves) for long (small wave number) waves [10-13].
- Effective numerical solver for modeling surface gravity waves was developed and it was verified by numerical experiment that the statistical steady state of surface gravity waves correspond to predictions of wave turbulence theory (Zakharov-Filonenko spectrum) [13].
- Strong intermittency was observed in our numerical experiments: Probability of having long waves of large amplitude is anomalously high. We have observed ten-fold increase of probability of large amplitude waves in comparison with Gaussian values. This observation sheds light on the processes of formation of freak waves in the ocean [10].

This year results:

- Previous derivations of the kinetic equation for internal waves were analyzed. It was found that because of small displacement assumptions previous formulations greatly overemphasized the role of scale-separated wave-wave interactions [14].
- *Solvability condition* that determines which member of the family of steady state solutions is realized in the case when ocean rotation is the only relevant perturbation to the otherwise scale invariant solution of the internal wave kinetic equation. It is demonstrated that such selection condition chooses the celebrated Garrett-and-Munk spectrum as the steady solution [14].
- Detailed analysis of the previously published data on the internal waves observation is summarized and submitted for publication [14].
- Numerical experiments performed on the dynamical equations for surface gravity waves: phases and amplitudes are indeed independent random variables, as predicted by the theory. However, unlike theory predicts, there are two types of waves for each Fourier mode: one with the frequency predicted by the linear dispersion law and the other, much higher, frequency. This observation has significant implications for surface gravity waves, as well as internal waves in the ocean [13].

IMPACT/APPLICATIONS

Continuing results from this project will significantly enhance our understanding of nonlinear wave interactions in shallow- and deep-water environments and consequently will lead to improved forecasting and prediction for Naval and civilian applications.

TRANSITIONS

RELATED PROJECTS

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PUBLICATIONS

All publications resulting from this project could be downloaded from my web-page, http://www.rpi.edu/~lvovy/.

•Published: [5-12].

•Submitted: [13,14]